Definition: A polynomial is an algebraic expression that consists of a finite sum of terms of the form ax^n, where a is a real number and n is a whole number. The standard form is to write the polynomials so that the degrees of the terms are in decending order.

Example: $-3x^4+2x^2-5x^1+7x^0$ is a polynomial

$\frac{2}{x}$, $6x^{-2}+4x^{-1}$, $\sqrt{y^2+3}$ are NOT polynomials.

Definition: A polynomial function is a function of the form

$$P(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x^1+a_0x^0$$

Where $a_n, a_{n-1}, \ldots, a_1$, and a_0 are real numbers and n is a whole number.

Adding and Subtracting Polynomials

Like Terms	Unlike Terms
$-a$, $\frac{1}{2}a$	$3y^2$, $3y^1$
$\frac{2x^2}{3}$, $-x^2$	$4x$, 7

$6x^2+3x^2$ because these are like terms
$=(6+3)x^2$ we add $6+3$
$=9x^2$

Procedure: Adding Polynomials

Step 1: Remove the parentheses
Step 2: Group like terms together
Step 3: Combining like terms
Step 4: Write the answer in standard form

$(x^2+2x+3)+(3x^2-x-1)$ (because the opperation is addition the parentheses are not needed)
$= x^2+3x^2+2x-x+3-1$
$= (1+3)x^2+(2-1)x+2$
$= 4x^2+x+2$

Procedure: Subtracting Polynomials

Step 1: Find the opposite of the polynomial that is being subtracted
Step 2: Combine like terms
Step 3: Write the answer in standard form.

$(2x^2-3x+5)-(x^2+2x-1)$
$= 2x^2-3x+5-x^2-2x+1$ distribute the negative
$= 2x^2-x^2-3x-2x+5+1$
$= (2-1)x^2-(3+2)x+6$
$= x^2-5x+6$

Multiplying Monomials and Polynomials

Property	Example
$a^m a^n = a^{m+n}$	$2x(5x) = 2\cdot5\cdot x^1\cdot x^1 = 10x^{1+1} = 10x^2$
$a(b+c) = ab+ac$	$5(x+4) = 5x+5(4) = 5x+20$

Procedure: Multiplying a Monomial by a Polynomial

Step 1: Distribute the monomial to each term of the polynomial
Step 2: Multiply the coefficients and multiply any like bases by adding the exponents.

Examples:

$$5x(x+4)$$
$= 5x(x) + 5x(4)$ Apply the distributive property
$= 5x^{1+1} + 20x$ Simplify each product
$= 5x^2 + 20x$

Multiplying Polynomials and Polynomial functions

Examples:

$$(x+3)(x^2+x+6)$$
$$= x(x^2+x+6) + 3(x^2+x+6)$$
$$= x(x^2) + x(x) + x(6) + 3(x^2) + 3(x) + 3(6)$$
$$= x^3 + x^2 + 6x + 3x^2 + 3x + 18$$
$$= x^3 + x^2 + 3x^2 + 6x + 3x + 18$$
$$= x^3 + 4x^2 + 9x + 18$$

$$(x-2)(x^2+2x+4)$$
$$x(x^2+2x+4) \Rightarrow x^3 + 2x^2 + 4x$$
$$-2(x^2+2x+4) \qquad -2x^2 - 4x - 8$$
$$x^3 + 0x^2 + 0x - 8$$
$$= x^3 - 8$$